Interaction Terms in Regression Models

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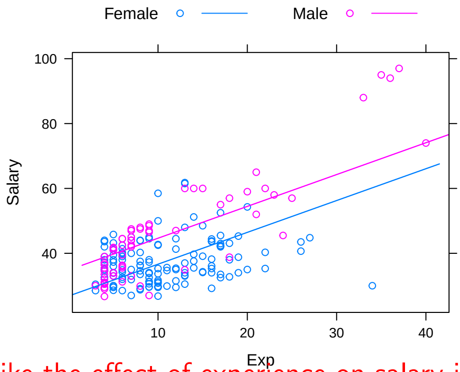
Back to the Salary Discrimination Case

We left off by fitting the model:

$$Salary_i = \beta_0 + \beta_1 Male_i + \beta_2 Exp_i + \epsilon_i$$



Does it look like the effect of experience on salary is the same for males and females?

Back to the Salary Discrimination Case

Could we try to expand our analysis by allowing a different slope for each group?

Yes! Consider the following model:

$$Salary_i = \beta_0 + \beta_1 Exp_i + \beta_2 Male_i + \beta_3 Exp_i \times Male_i + \epsilon_i$$

For Females:

$$Salary_i = \beta_0 + \beta_1 Exp_i + \epsilon_i$$

For Males:

$$Salary_i = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) Exp_i + \epsilon_i$$

Salary Discrimination Case

What do the data look like?

	Exp	Gender	Salary	Male	Exp*Male
1	3	Male	32.00	1	3
2	14	Female	39.10	0	0
3	12	Female	33.20	0	0
4	8	Female	30.60	0	0
5	3	Male	29.00	1	3
...	...	...			
208	33	Female	30.00	0	0

`R` will make the dummy variable Male and the interaction term Exp*Male for us

Salary Discrimination Case

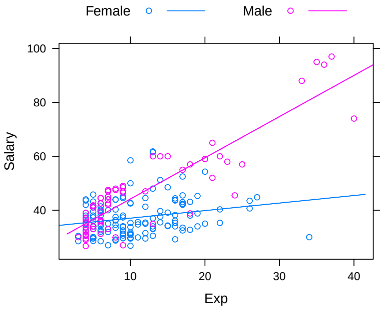
```
salaryfit_int = lm(Salary~Gender*Exp, data=salary)
```

```
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)       34.2483     1.2274  27.903  < 2e-16 ***
## GenderMale        -5.3461     1.7766  -3.009  0.00295 **
## Exp                0.2800     0.1025   2.733  0.00684 **
## GenderMale:Exp     1.2478     0.1367   9.130  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.816 on 204 degrees of freedom
## Multiple R-squared:  0.6386,Adjusted R-squared:  0.6333
## F-statistic: 120.2 on 3 and 204 DF,  p-value: < 2.2e-16
```

Is this good or bad news for the plaintiff?

Salary Discrimination Case

In our new model the gender gap in wages is different depending on the experience of the employee:



The expected male salary minus the expected female salary is:

$$\beta_2 + \beta_3 Exp$$

Salary Discrimination Case

We can report estimates & confidence intervals for the wage gap at different levels of experience using the bootstrap:

```
gap1 = do(1000) * {
  fit = lm(Salary ~ Gender * Exp, data = resample(salary))
  betas = coef(fit)
  exper = 5 # 25th percentile of experience
  betas[2] + betas[4]*exper
}

confint(gap1)

##         name     lower    upper level     method  estimate
## 1 GenderMale -1.206776 3.165836  0.95 percentile 0.8929416
```

Salary Discrimination Case

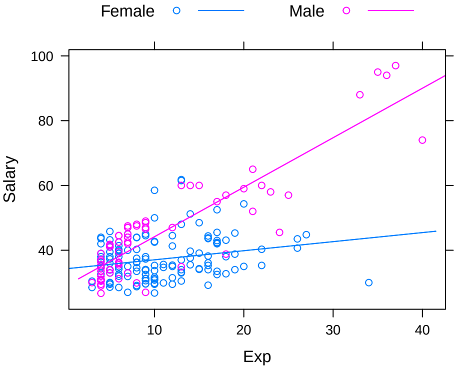
We can report estimates & confidence intervals for the wage gap at different levels of experience using the bootstrap:

```
gap2 = do(1000) * {
  fit = lm(Salary ~ Gender * Exp, data = resample(salary))
  betas = coef(fit)
  exper = 10
  betas[2] + betas[4]*exper
}

confint(gap2)

##         name    lower    upper level     method estimate
## 1 GenderMale 5.206036 9.097281  0.95 percentile 7.131933
```

Salary Discrimination Case

$$Salary = \beta_0 + \beta_1 Male + \beta_2 Exp + \beta_3 Exp * Male + \epsilon$$

```
plotModel(salaryfit_int, Salary~Exp)
```



$$Salary = 34 - 4 Male + 0.28 Exp + 1.24 Exp * Male + \epsilon$$

Interaction Terms vs Group-Specific Models

We could have gotten similar results by fitting linear models for males and females separately. Why use interaction terms?

If we want to include other variables with effects on salary that **don't** differ by gender – for example:

$$Salary = \beta_0 + \beta_1 Male + \beta_2 Exp + \beta_3 Exp * Male + \beta_4 College + \epsilon$$

we can't use data subsetting.

Interaction terms are the exception, not the rule!

Interaction Terms: Other Cases

We can interact two continuous variables too:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 (X_1 X_2) + \varepsilon$$

Fixing X_1 at d we have a line in Y and X_2:

$$Y = (\beta_0 + \beta_1 d) + (\beta_2 + \beta_3 d) X_2 + \varepsilon$$

So the effect of a unit increase in X_2 on the predicted value of Y, holding X_1 constant – given by the slope above – depends on d, the actual value where we hold X_1 constant.

To interact two categorical variables, it's easiest to make one combined categorical variable that takes each possible comination of the original two variables

Example: College GPA and Age

Consider the relationship between undergrad and MBA grades:

A model to predict McCombs GPA from undergrad GPA could be

$$GPA^{MBA} = \beta_0 + \beta_1 GPA^{Bach} + \beta_2 Age + \varepsilon$$

```
fit1 = lm(MBAGPA ~BachGPA+Age, data=gpa)
coef(fit1)

## (Intercept)     BachGPA         Age
##  3.66400247  0.19495068 -0.03024069
```

For every 1 point increase in college GPA, expected MBA GPA increases by about .19 points, holding constant age.

College GPA and Age

Assumes that the partial effect
of College GPA is the same for any age.

It seems that how you did in college should have less effect on your MBA GPA as you get older (farther from college).

We can account for this intuition with an interaction term:

$$GPA^{MBA} = \beta_0 + \beta_1 GPA^{Bach} + \beta_2 Age + \beta_3 (Age \times GPA^{Bach}) + \varepsilon$$

Now the effect of a one-point increase in undergrad GPA holding constant Age is $\beta_1 + \beta_3 Age$.

Depends on the value of Age!

College GPA and Age

```r
lm(MBAGPA ~ BachGPA*Age, data=gpa)

##
## Call:
## lm(formula = MBAGPA ~ BachGPA * Age, data = gpa)
##
## Coefficients:
## (Intercept)      BachGPA          Age  BachGPA:Age
##    -0.27964      1.36936      0.10974     -0.04181
```

College GPA and Age

Without the interaction term

- Effect of College GPA controlling for Age is $\hat{\beta}_1 = 0.26$.

With the interaction term:

- Effect is $\hat{\beta}_1 + \hat{\beta}_3 Age = 1.37 - 0.042 Age$.

Age	Marginal Effect
24	0.36
27	0.24
30	0.11

Interactions: Things to remember

You should almost never try to interpret/test the main effect of a variable involved in an interaction. (You can't hold the interaction constant and vary the main effect!)

Usually if an interaction between two variables is present you should include both main effects. (i.e., if X_1X_2 is a term in your model you should also include X_1 and X_2 terms)

When do I need an interaction term?

In MLR, if you want to study the effect of one variable (Gender) on another (Salary) while **controlling for** (or **holding constant**) a third (Experience), you don't usually need an interaction.

You only need an interaction term if the relationship between the two variables (Gender and Salary) **depends on the specific value of the other variable** (Experience)

In our first example, it seemed like the wage gap was larger for more experienced workers, which suggested an interaction term.