Expected Value and Variance

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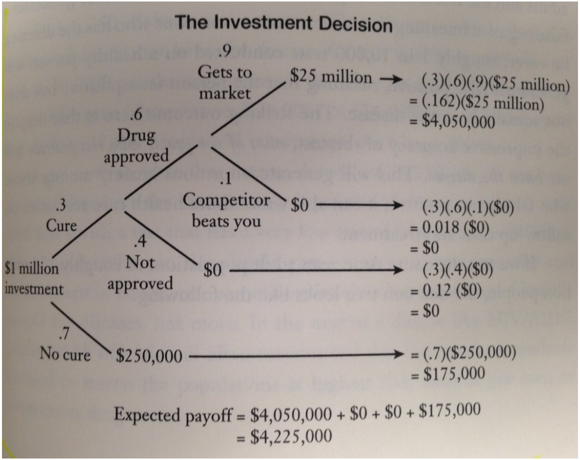
Probability and Decisions

- So you've tested positive for a disease. Now what?
- Let's say there's a treatment available. Do you take it?
- What additional information (if any) do you need?
- We need to understand the **probability distribution** of outcomes to assess **(expected) returns** and **risk**

Example: Drug Investment

You are presented with the opportunity to invest in the development of a drug... should you do it?



Example: Drug Investment

We have a random variable, our revenue, with the following probabilities...

Revenue	$P(Revenue)$
\$250,000	0.7
\$0	0.138
\$25,000,000	0.162

Should we invest? How much would be reasonable to invest?

Probability and Decisions

What if the distribution of revenue looked like this instead?

Would you prefer this investment?

Revenue	$P(Revenue)$
\$3,721,428	0.7
\$0	0.138
\$10,000,000	0.162

Expected Value and Variance of a Random Variable

The Expected Value (or mean) of a random variable X is defined as (for a discrete X with n possible outcomes):

$$E(X) = \sum_{i=1}^{n} Pr(X = x_i) \times x_i$$

We weight each possible value by how likely they are... this provides us with a measure of centrality of the distribution and a "good" prediction for X.

Example: Mean and Variance of a Binary Random Variable

Suppose

$$X = \begin{cases} 1 & \text{with prob.} \quad p \\ 0 & \text{with prob.} \quad 1-p \end{cases}$$

$$E(X) \quad = \quad ?$$

Mean and Variance of a Random Variable

The Variance is defined as (for a discrete X with n possible outcomes):

$$Var(X) = \sum_{i=1}^{n} Pr(X = x_i) \times [x_i - E(X)]^2$$

Weighted average of squared prediction errors... This is a measure of spread of a distribution. More risky/unpredictable distributions have larger variance.

Example: Mean and Variance of a Binary Random Variable

Suppose

$$X = \begin{cases} 1 & \text{with prob.} \quad p \\ 0 & \text{with prob.} \quad 1-p \end{cases}$$

$$Var(X) \quad = \quad ?$$

Question: For which value of p is the variance the largest? For which value of p is the outcome **least** predictable?

The Standard Deviation

- What are the units of $E(X)$? What are the units of $Var(X)$?
- A more intuitive way to understand the spread of a distribution is to look at the standard deviation:

$$sd(X) = \sqrt{Var(X)}$$

- What are the units of $sd(X)$?

Mean, Variance, Standard Deviation: Summary

What to keep in mind about the mean, variance, and SD:

- The expected value/mean is usually our **best prediction** of an uncertain outcome. ("Best" meaning closest in distance to the realized outcome, using a particular measure of distance)
- The variance is often a reasonable summary of **how unpredictable** an uncertain outcome is (or how risky it is to predict)
- The standard deviation (square root of the variance) is another reasonable summary of risk/unpredictability that is on a meaningful scale.