STA-371G
Homework 6

This assignment contains 2 questions; there are 77 total points.

Submit your assignment at the **beginning** of class on 4/3/2019. Please make sure that the pages of your homework assignment are in the correct order and securely stapled together. You may (but are not required to) work in groups of up to four students. Each group should hand in one copy of the homework.

Grading

A small number of problems will be graded for correctness, and the remainder graded for completeness. A complete response answers the question posed and also shows your work. This means showing the steps of a mathematical calculation, or including R output that justifies your conclusions. For questions that are not just calculations (e.g., more than computing an expected value from a table) you should answer in complete, concise sentences. Detailed solutions will be available – you should always check your work against these solutions.

---

1. (52 points) Read the "Milk and Money" case in the course packet. The data for this case are available from the course website, and you can read them into R using the code below (also available in the R script posted with this assignment).

```
# Run this code to load the data into your R session.
# Or visit the website and download the code, and then import. This
# is only necessary if you want to work offline. You'll see a bunch of red text
# after running the code, but if it matches what you see below
# (starting from ## Parsed with column specification... )
# it's nothing to worry about.
library(readr)
path = "https://jaredsmurray.github.io/sta371g_s19/data/"
milk = read_csv(paste0(path, 'milk.csv'))
```

Important information:

1. The Federal government, through the Agricultural Marketing Service (AMS), sets the price that dairy farmers receive for different "classes" of milk (these classes are called Class I, Class II, etc.). The prices set by the AMS depend on, among other things, the wholesale price of milk and can vary significantly over a two or three year period. In this problem, we will be concerned only with Class III milk prices.

2. A farmer can purchase a put option that gives him the right but not the obligation to sell a futures contract on Class III milk at the "strike" price on or before the expiration date of the option. This puts a "floor" under the price that the farmer will receive for his Class III milk. He removes the downside risk but still has the upside potential.

   For example, suppose the strike price on a December 15 Class III milk put option is \$12/cwt (cwt is a unit of measurement that is roughly 100 pounds of milk). If the AMS price on December 15 is below \$12/cwt, the put option allows the farmer to sell his milk for \$12/cwt. If the AMS price is greater than \$12/cwt then he will sell his milk at the AMS price.

   The cost of the put option is the price a farmer must pay someone to take on the downside risk. For example, the cost of a \$12/cwt December 15 put option purchased in June might be \$0.45/cwt.

   The farmer must also pay trading costs for purchasing the option (e.g. broker?s commission, etc.). For example, the trading cost on a \$12/cwt December 15 option might be \$0.05/cwt.

   Strike prices on put options for Class III milk are available every \$0.25. For example, \$11.50/cwt, \$11.75/cwt, \$12/cwt, \$12.25/cwt, etc.

3. For historical and legal reasons, California dairy farmers participate in a California pricing system rather than the federal AMS pricing system. The price a California dairy farmer receives for his milk, called the "mailbox" price, is determined by a complex formula that depends on the value of various dairy products on the wholesale market. The California mailbox price varies a great deal over time just as the federal AMS price does. For example, between 2005 and 2007 the mailbox price varied between \$10.16/cwt and \$19.98/cwt with an average price in 2006 of \$11.28/cwt. The dairy farmer in the case, Gerard, estimates his costs are \$12/cwt so a price of \$11.28/cwt creates a significant financial problem for him.

4. Gerard is interested in hedging his revenue six months in advance and guaranteeing a price of at least \$12/cwt for his milk. For example, in June he wants to hedge his December 15 revenue.

5. Put options on the California mailbox price are not available. The federal Class III milk price is closely related, although not the same as, the California mailbox price that Gerard will receive. For this reason, Gerard will use put options on the federal Class III milk price to hedge his revenue.

6. Gerard wants the probability to be at least 95% that his revenue will be \$12/cwt or more no matter what the California mailbox price is.

Parts (a) and (b) below (with solutions in blue) provide an example of how to determine the value of a put option on Class III milk on its expiration date. The same idea is used in a slightly more complicated context in parts (c) – (j).

The discussion of put options on pages 6-8 of the case, and in particular the example at the bottom of page 7 and top of page 8 will be helpful in answering parts (a) and (b).

(a) Suppose Gerard buys a December 15 put option on Class III milk in June with a strike price of \$12/cwt. If the Class III milk price on December 15 is \$11.50/cwt, how much is the put option worth when it expires on this day?

The put option is worth \$0.50. The reason is that the option allows the holder to sell Class III milk for \$12/cwt on December 15 while the price of Class III milk through the AMS government program is only \$11.50/cwt. This means the option provides the holder with an additional \$0.50/cwt in revenue that he would not receive if he sold his milk through the government program. Therefore, someone would be willing to pay up to \$0.50 to purchase the option on the day it expires.

(b) Suppose the price for the put option in part (a) is \$0.30/cwt and that the trading costs for purchasing the option are \$0.05/cwt. Combining the value of the option obtained in part (a) with the cost information, what is Gerard's net gain on the option (i.e. what is the value of the put option minus the option cost and trading cost)? Gerard's net gain is \$0.15. This includes the \$0.50 he makes on the put option minus the \$0.30 premium paid for the option minus the \$0.05 paid in trading costs. Therefore, his net gain is \$0.50 - (\$0.30 + \$0.05) = \$0.15.

For parts (c)–(j), suppose Gerard in June decides to hedge his December 15 revenue by purchasing a put option on Class III milk with a strike price of \$14.25/cwt and an expiration date of December 15.

You should do the calculations for parts (c)–(h) assuming the costs of the option are zero. A way to incorporate the additional costs into the hedging process is discussed in parts (i) and (j).

(c) (15 points) Plot the Class III milk price against the California mailbox price and add the estimated regression line to the plot. What is the equation of the estimated regression line? How much do you expect the Class III milk price to change on average for a \$1/cwt change in the California mailbox price?

For the remainder of the problem use 0.60 as the value of σ (the standard deviation of the error term) and the estimated regression line as if it were the true regression line to answer the following questions.

(d) (5 points) What is the probability that Gerard's December 15 put option on Class III milk with a strike price of \$14.25 is in the money (i.e. worth something) on December 15 if the California mailbox price on December 15 is \$12.50/cwt?

(e) (5 points) Suppose the California mailbox price on December 15 is \$12.00/cwt. What is the probability that the value of the put option will be greater than \$0.50?

(f) (5 points) Using your answer to part (e), what is the probability that Gerard's net revenue (mailbox price plus payoff from the option) will exceed \$12.50/cwt if the California mailbox price is \$12.00/cwt?

(g) (5 points) Now suppose the mailbox price on December 15 is \$11.50/cwt. What is the probability that the value of the put option will be greater than \$1? Using your answer to this question, what is the probability that Gerard's net revenue (mailbox price plus payoff from the option) will exceed \$12.50/cwt?

(h) (7 points) Is the probability at least 95% that his net revenue (mailbox price plus payoff from the option) will equal or exceed \$12.50/cwt for any mailbox price below \$12.50/cwt? Why or why not?

For parts (i) and (j), suppose the price in June for a December 15 put option on Class III milk with a strike price of \$14.25/cwt is \$0.45/cwt and the trading cost is \$0.05/cwt.

(i) (5 points) Is the probability at least 95% that Gerard's net revenue (mailbox price plus payoff from the option minus option cost and trading cost) will equal or exceed his production costs of \$12.00/cwt no matter what the California mailbox price is?

(j) (5 points) Has Gerard effectively hedged his net revenue (mailbox price plus payoff from the option minus option cost and trading cost) if \$12/cwt is the amount he needs to receive for his milk to cover his production costs?

2. (25 points)

Revisit the case "Waite First Securities" in the course packet.

In the last homework you fit the models

$$Hilton_t = \beta_0 + \beta_1 SP500_t + \epsilon_t$$

$$Giant_t = \beta_0 + \beta_1 SP500_t + \epsilon_t$$

$$TI_t = \beta_0 + \beta_1 SP500_t + \epsilon_t$$

(a) (15 points) For each of the three stocks provide the p-value associated with the null hypothesis that the stock is just as risky as the market. For which stock is the evidence against the null hypothesis strongest? Weakest?

(b) (10 points) Describe in your own words why it would be preferable here to report the estimate and its standard error, or equivalently an estimate and confidence interval, instead of the p-value.