

# Homework 4

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## Instructions

Submit your assignment at the **beginning** of class on **Weds Mar 6**. Please make sure that the pages of your homework assignment are in the correct order and securely stapled together. You may (but are not required to) work in groups of up to four students. Each group should hand in one copy of the homework.

## Grading

A small number of problems will be graded for correctness, and the remainder graded for completeness. A complete response answers the question posed and also shows your work. This means showing the steps of a mathematical calculation, or including R output that justifies your conclusions. For questions that are not just calculations (e.g., more than computing an expected value from a table) you should answer in complete, concise sentences. Detailed solutions will be available – you should always check your work against these solutions.

## Problem 1 (10 points)

A manufacturer of automobile batteries claims that the distribution of the lifetimes of its best battery has a mean of 54 months and a standard deviation of 6 months. Suppose a consumer group decides to check the claim by purchasing a sample of 50 of these batteries and subjecting them to tests that determine their lifetimes. Assuming the manufacturer's claim is true, what is the probability that the consumer group's sample has a mean lifetime of 52 months or less?

## Problem 2 (10 points)

For the SP500 daily returns example on slides 2-7 of section 1.4, we estimated the average daily return  $\mu$  using the sample mean return, which was  $\bar{X} = 0.03\%$ . Compute a 95% confidence interval for the average daily returns of the SP500. Is 0.15% a plausible value for the average daily return? Why or why not?

## Problem 3 (10 points)

McDonald's recently administered a survey to a random sample of 470 of its franchisees. There are about 12,900 franchisees in the U.S. They found that 35% of respondents said

that they were confident in McDonald's long term success ( $\hat{p} = 0.35$ ). Calculate a 95% confidence interval for the same percentage in the population.

#### **Problem 4 (34 points)**

Read the "Amore Frozen Foods, UVA-QA-0317" case in the course packet.

The primary question to answer in the case is: Should Mr. Jenkins lower the fill target to 8.22 ounces per pie from the current target of 8.44 ounces per pie?

The information given below is obtained from the case and will be helpful in answering the above question.

##### **Information given in the case:**

1. The filling device fills to target with a standard deviation of 0.22 ounces and the amounts are normally distributed.
2. The five sample pies' weights are independent.
3. The production rate is 1,000 dozen pies every 20 minutes.
4. The monthly production is 60 20-minutes batches so annual production is  $60 \times 12 = 720$  batches.
5. Rejected batches are first sold at the Thrift Store and then donated to charity once the Thrift Store demand has been filled.
6. The Thrift Store demand is 60 dozen pies per week.
7. The price of pies sold at wholesale is \$4.50 per dozen.
8. The price of pies sold at the Thrift Store is \$3.60 per dozen.
9. The cost of ingredients at the 8.44 ounce target fill rate is \$1.82 per dozen pies. The cost of ingredients at the 8.22 ounce target fill rate is proportional (i.e.  $(8.22/8.44) \times \$1.82 = \$1.77$  per dozen pies).
10. The cost of packaging is \$0.62 per dozen pies.
11. The cost of labor and overhead is \$0.56 per dozen pies.

##### **FDA Approved Weight Control System**

The FDA approved weight control system for Amore Frozen Foods specifies that a 20 minute batch must be rejected if the average of five sample pies taken at the beginning of the batch run is less than eight ounces.

*If the fill target is 8.22 ounces, answer the following questions:*

- (a) (2pts) What is the probability that a given batch will be rejected?

- (b) (2pts) How many batches over the course of a year do you expect to be accepted and how many do you expect to be rejected?
- (c) (2pts) What is the expected annual revenue that will be obtained from selling the accepted batches at wholesale?
- (d) (2pts) What is the expected annual revenue that will be obtained from selling pies at the Thrift Store?
- (e) (2pts) How many pies do you expect to be donated annually to charity?
- (f) (3pts) What are the annual ingredient costs? What are the annual packaging costs? What are the annual labor and overhead costs?
- (g) (4pts) What is the expected profit?

***If the fill target is 8.44 ounces, answer the following questions:***

- (h) (2pts) For practical purposes, why is the probability essentially zero that an individual batch will be rejected? (You should do a probability calculation to answer this question.)
- (i) (2pts) What is the expected annual revenue that will be obtained from selling the accepted batches at wholesale?
- (j) (2pts) What is the expected annual revenue that will be obtained from selling pies at the Thrift Store?
- (k) (2pts) How many pies do you expect to be donated annually to charity?
- (l) (3 pts) What are the annual ingredient costs? What are the annual packaging costs? What are the annual labor and overhead costs?
- (m) (4 pts) What is the expected profit?
- (n) (2 pts) What is your recommendation to Mr. Jenkins regarding the appropriate fill target? Why?

Table 1: Table To Summarize Annual Revenues, Costs and Profits For 8.22 and 8.44 Ounce Fill Targets

	8.22 Ounce Target	8.44 Ounce Target
Wholesale Revenue		
Thrift Store Revenue		
<b>Revenue</b>		
Ingredient Cost		
Packaging Cost		
Labor and Overhead Costs		
<b>Total Costs</b>		
<b>Profit</b>		