Using Bayesian Causal Forest Models to Examine Treatment Effect Heterogeneity

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Multilevel Linear Models for Heterogeneous Treatment Effects

School-specific intercepts/fixed/random effects

School-specific "unexplained" heterogeneity

$$y_{ij} = \alpha_j + \sum_{h=1}^{p} \beta_h x_{ijh} + \left[\sum_{\ell=1}^{k} \tau_\ell w_{ij\ell} + \gamma_j \right] z_{ij} + \epsilon_{ij}$$

Controls at the student and/or school level

Moderators at the student and/or school level

Coloring outside the lines: Multilevel Bayesian Causal Forests

We replace linear terms with Bayesian additive regression trees (BART)

$$y_{ij} = \alpha_j + \beta(\mathbf{x}_{ij}) + \left[\tau(\mathbf{w}_{ij}) + \gamma_j\right] z_{ij} + \epsilon_{ij}$$

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BART in causal inferece: Hill (2011), Green & Kern (2012), …

Parameterizing treatment effect heterogeneity with BART is due to Hahn, Murray and Carvalho (2017)

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Allows for complicated functional forms (nonlinearity, interactions, etc) without pre-specification…

…while carefully regularizing estimates with prior distributions (shrinkage toward additive structure and discouraging implausibly large treatment effects)

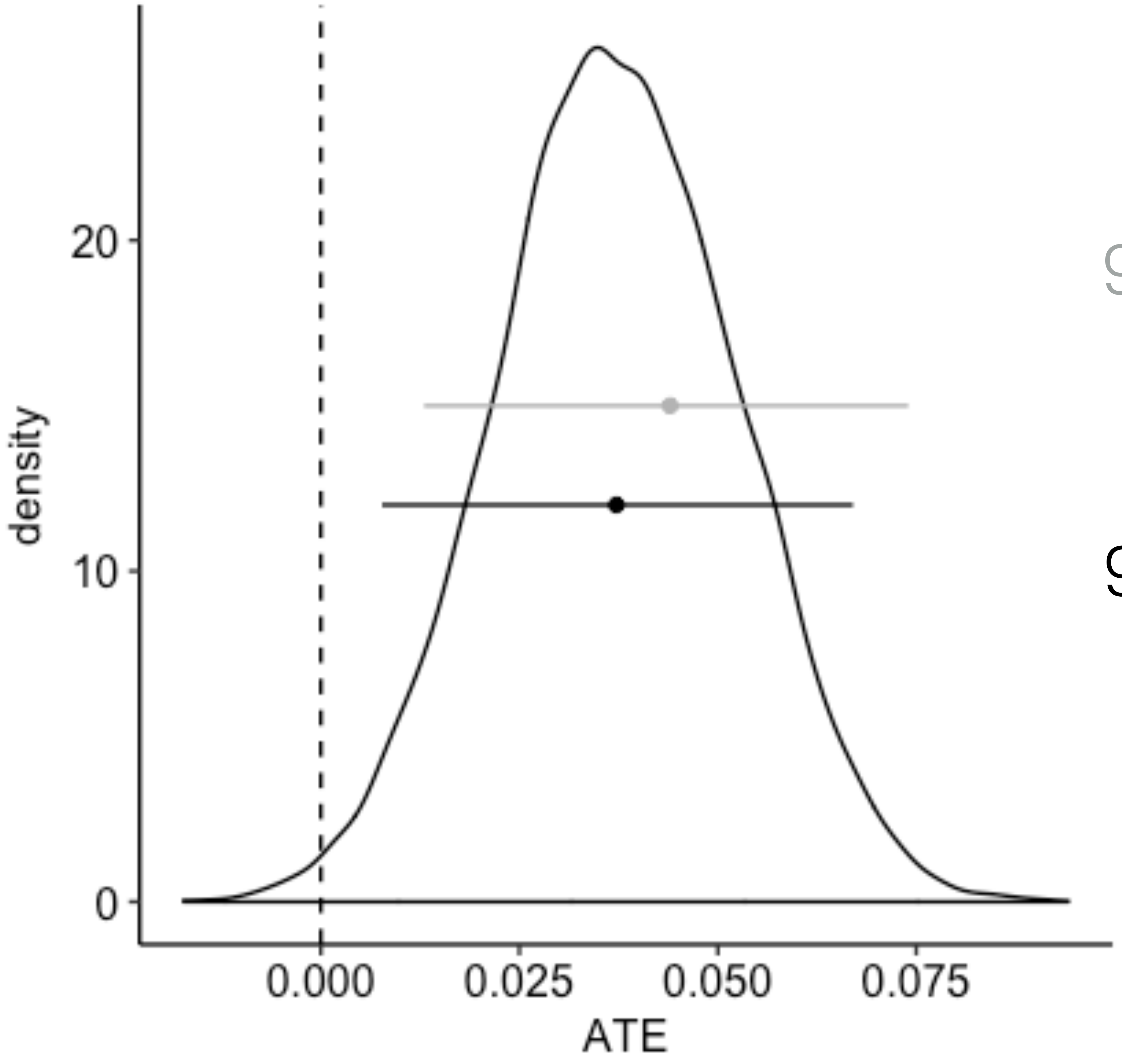
Analyzing data with ML BCF

- Obtain posterior samples for all the parameters, compute treatment effect estimates for each unit/school/etc.
- The challenge: How do we summarize these complicated objects?
    - “Roll up” treatment effect estimates to ATE
    - Subgroup search
    - Counterfactual treatment effect predictions/“partial effects of moderators”

Application: A new analysis with NMS

- Same moderators (school mindset norms, achievement, and minority composition) + controls
- Different population (all students) and outcome (math GPA)
- Same basic process with limited researcher DOF
    - Weakly informative priors on $\tau(w)$ (<0.5 GPA points with high prior probability) and random effects

Inference for the Average Treatment Effect

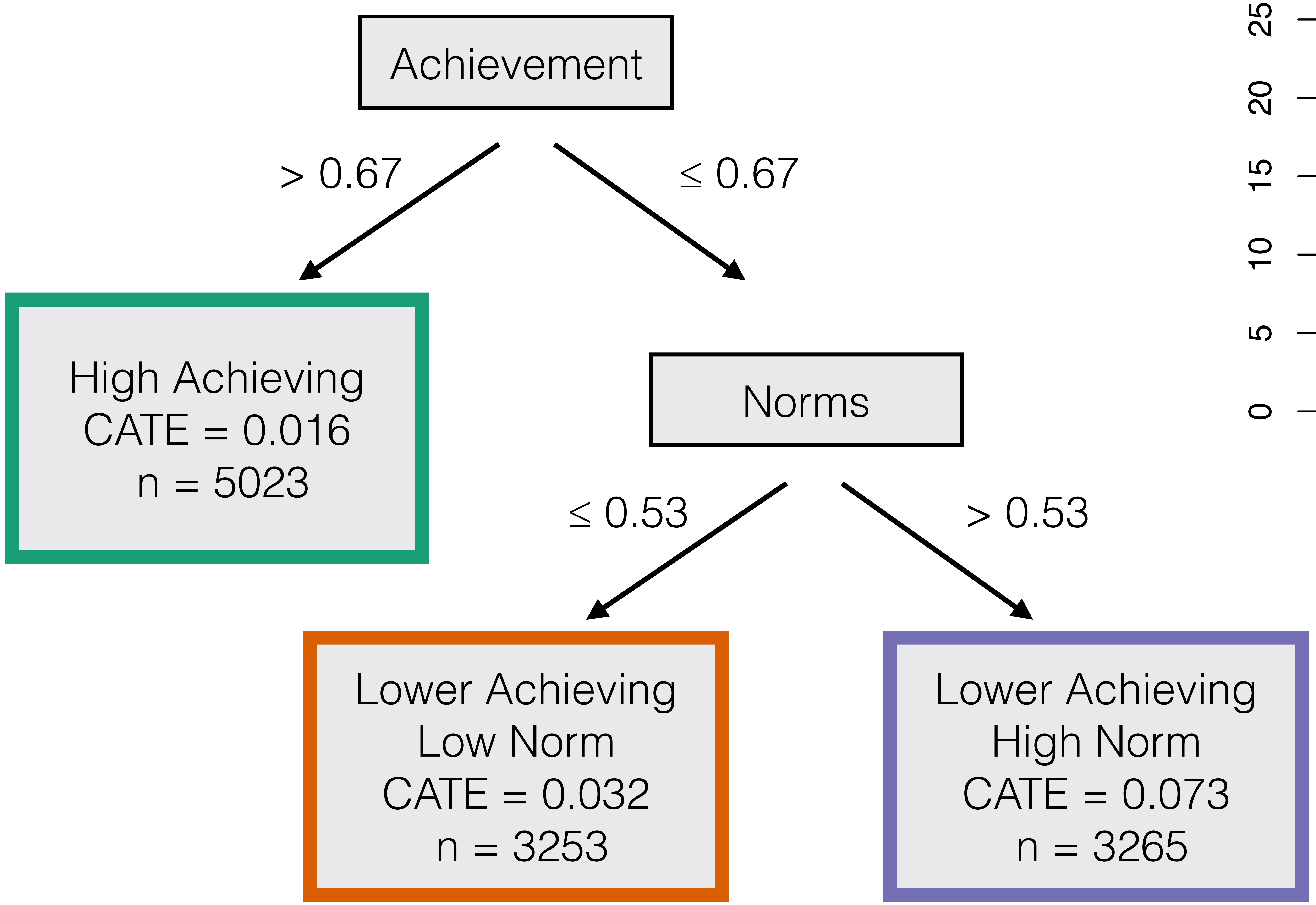


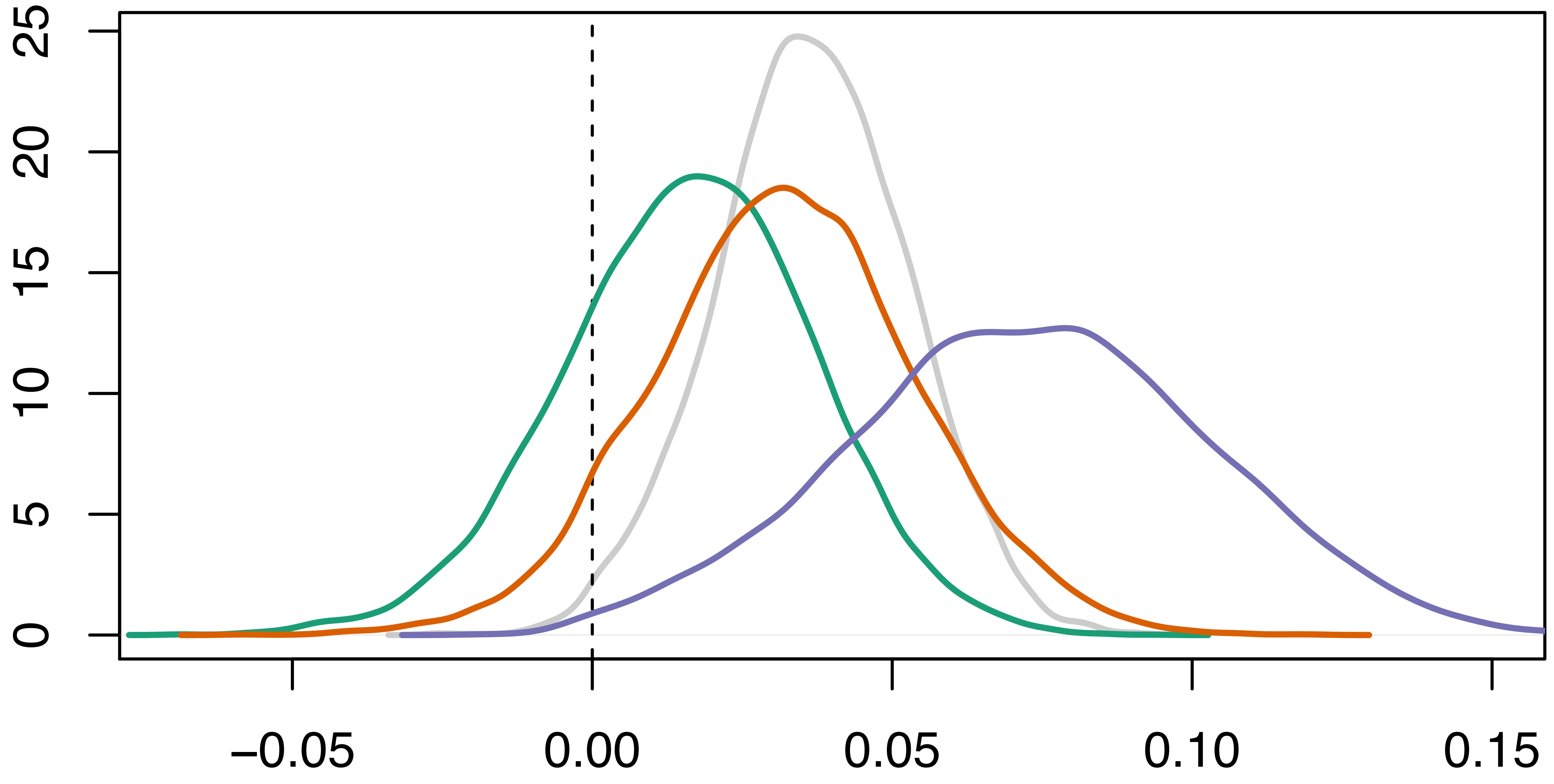
95% confidence interval from ML Linear Model

95% uncertainty interval from ML BCF

Subgroup search

- Obtain posterior mean of treatment effects
- Use recursive partitioning (CART) **on the posterior mean** to find moderator-determined subgroups with high variation across subgroup ATE
  - Statistically kosher! We use the data once (prior -> posterior)
  - Can be formalized as the Bayes estimate under a particular loss function


Achievement
> 0.67
≤ 0.67
High Achieving
CATE = 0.016
n = 5023
Norms
≤ 0.53
> 0.53
Lower Achieving
Low Norm
CATE = 0.032
n = 3253
Lower Achieving
High Norm
CATE = 0.073
n = 3265


0
5
10
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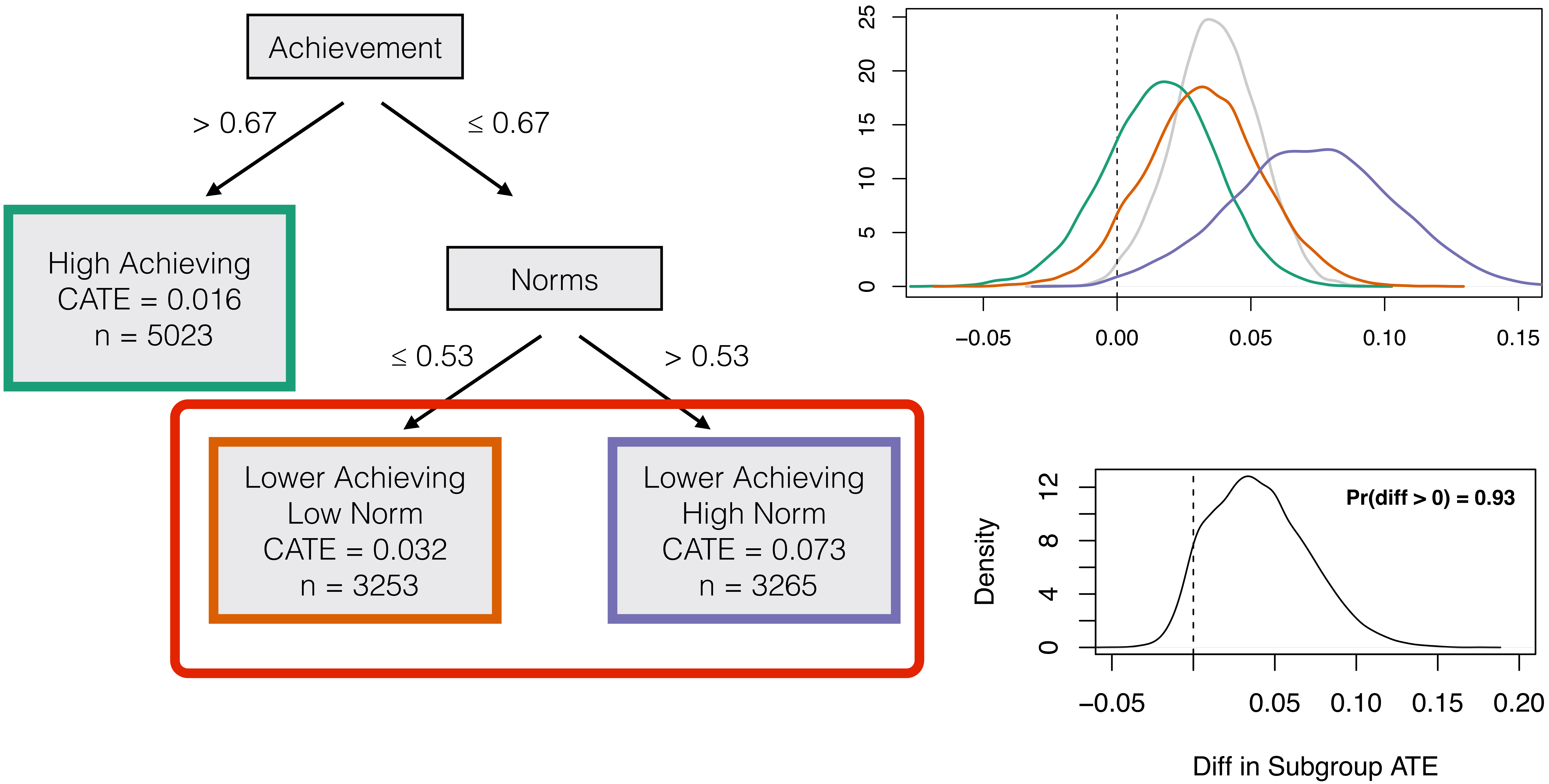
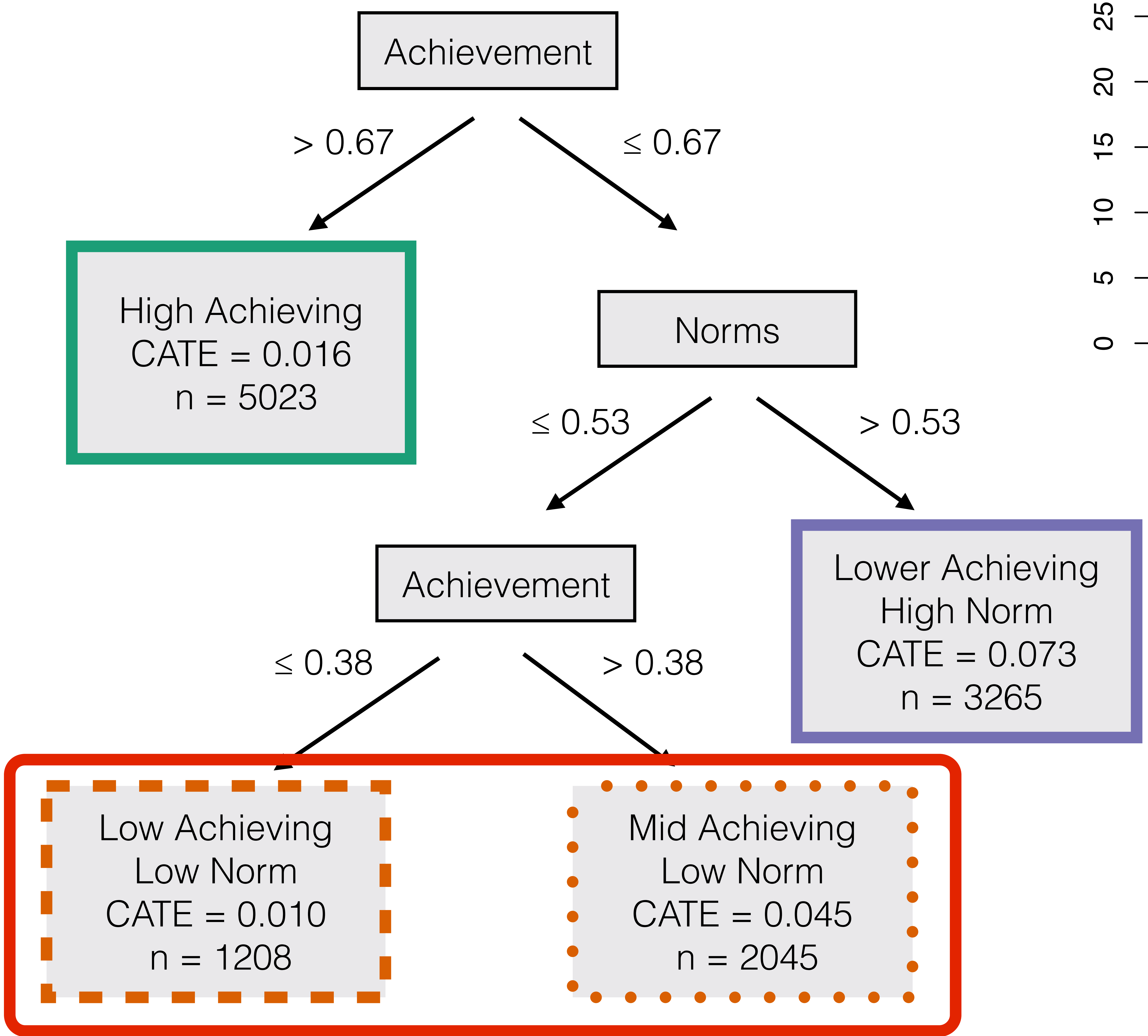
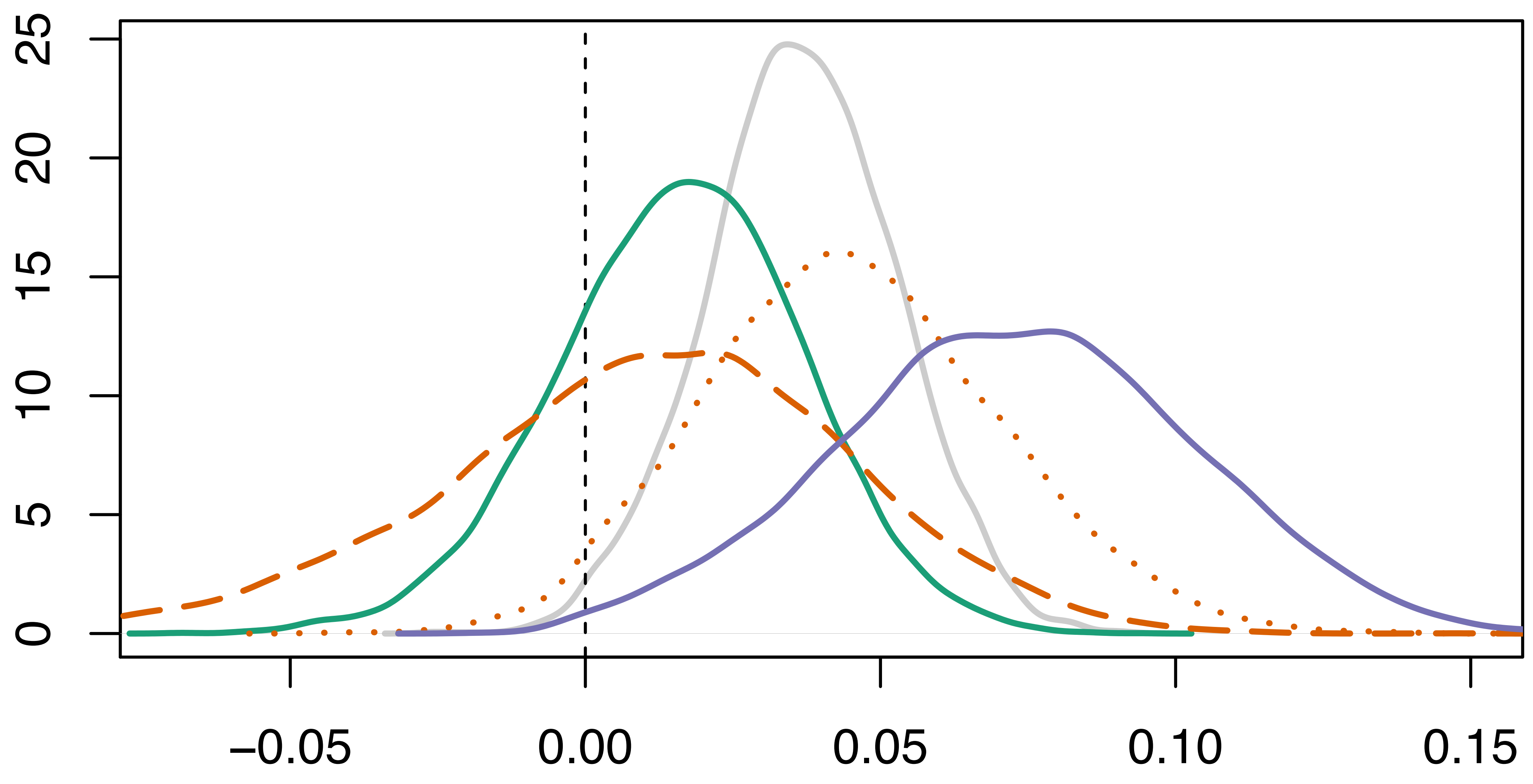
20
25
−0.05
0.00
0.05
0.10
0.15



Achievement
> 0.67
≤ 0.67
High Achieving
CATE = 0.016
n = 5023
Norms
≤ 0.53
> 0.53
Lower Achieving
Low Norm
CATE = 0.032
n = 3253
Lower Achieving
High Norm
CATE = 0.073
n = 3265
0
5
10
15
20
25
−0.05
0.00
0.05
0.10
0.15
Pr(diff > 0) = 0.93
Density
0
4
8
12
−0.05
0.05
0.10
0.15
0.20
Diff in Subgroup ATE


Achievement
> 0.67
≤ 0.67
High Achieving
CATE = 0.016
n = 5023
Norms
≤ 0.53
> 0.53
Achievement
Lower Achieving
High Norm
CATE = 0.073
n = 3265
≤ 0.38
> 0.38
Low Achieving
Low Norm
CATE = 0.010
n = 1208
Mid Achieving
Low Norm
CATE = 0.045
n = 2045


0
5
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−0.05
0.00
0.05
0.10
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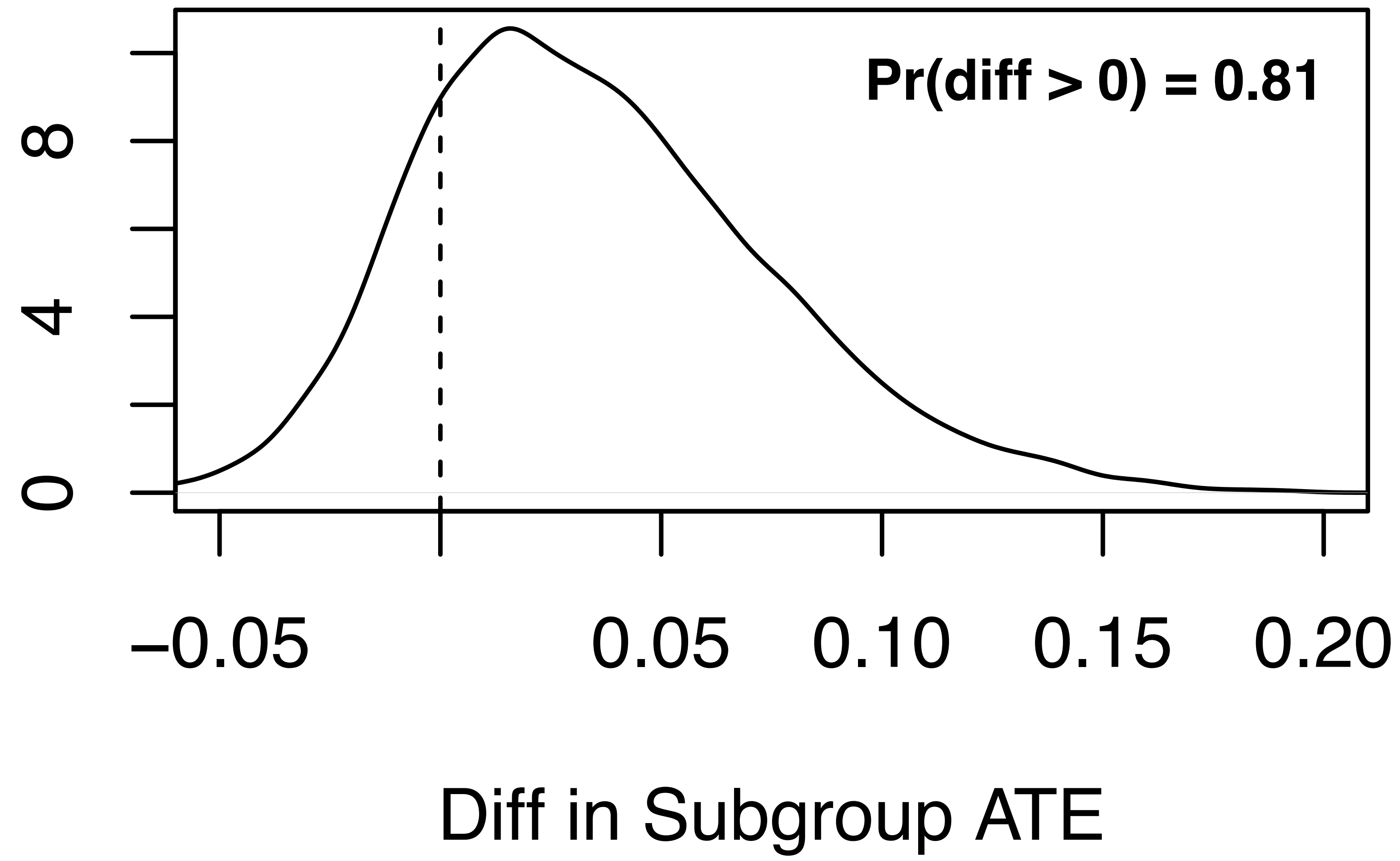
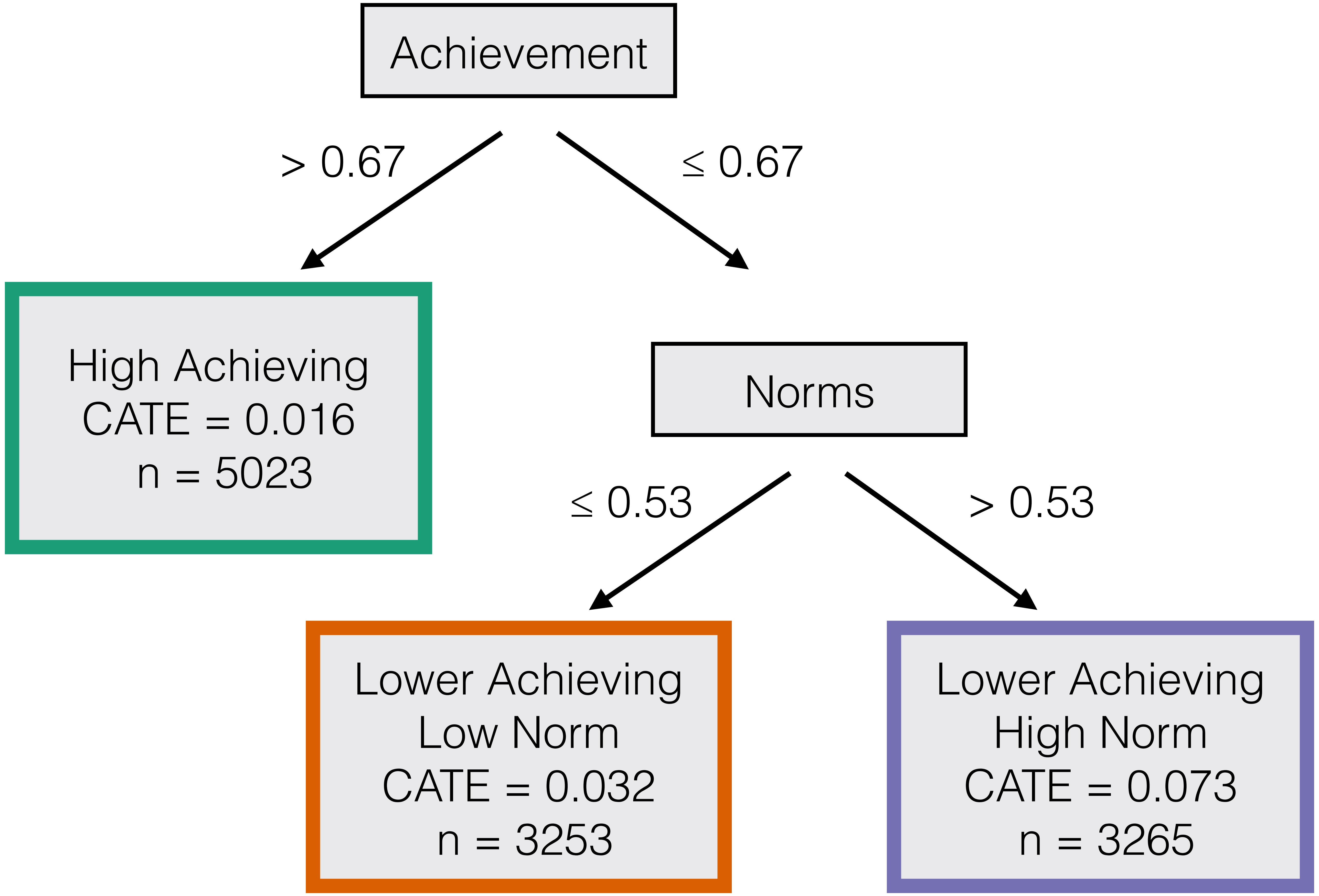
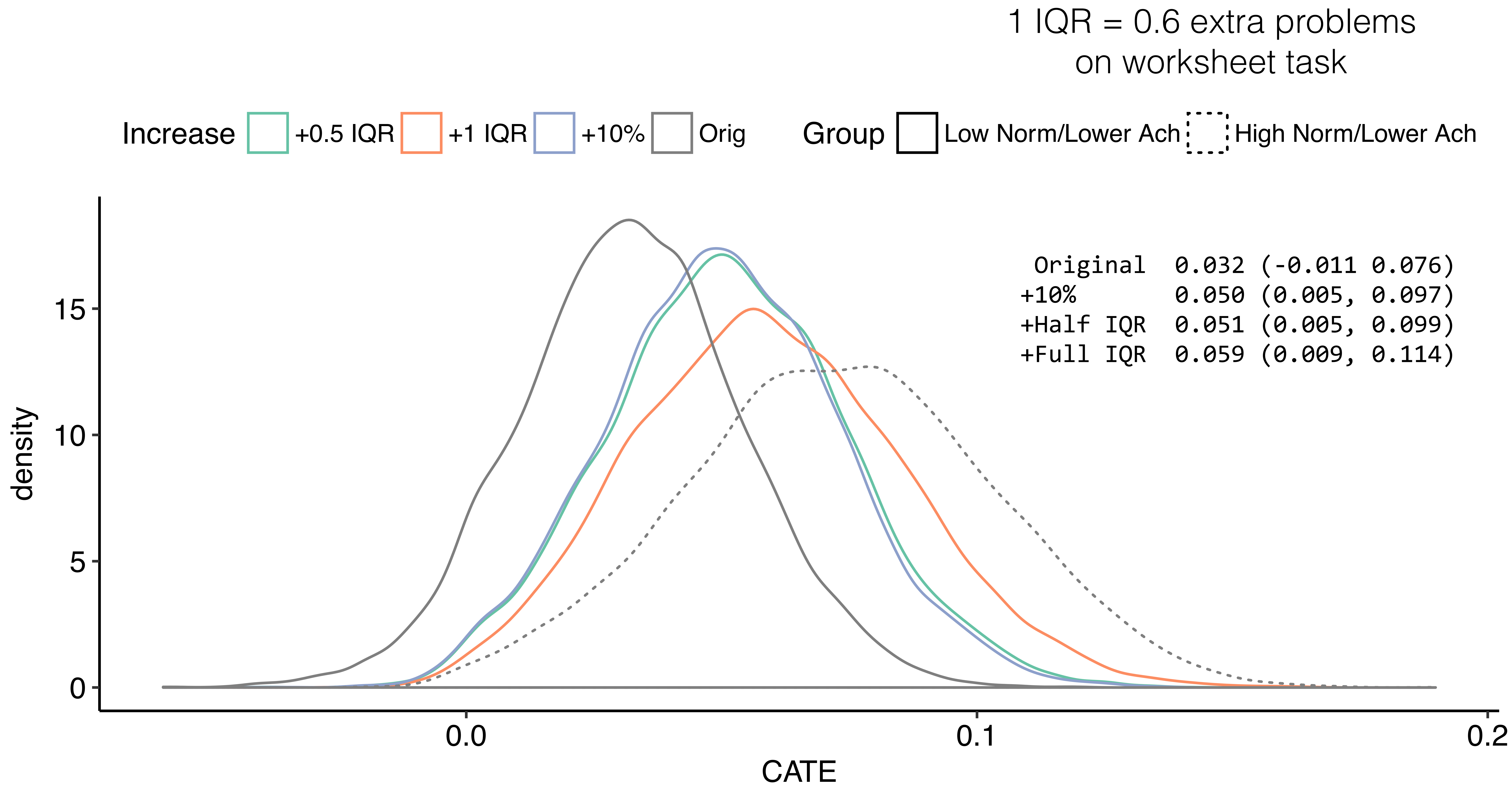

Pr(diff > 0) = 0.81
0
4
8
−0.05
0.05
0.10
0.15
0.20
Diff in Subgroup ATE

Counterfactual treatment effect predictions

- How do estimated treatment effects change in lower achieving/low norm schools if norms increase, holding constant school minority comp & achievement?
- Not a formal causal mediation analysis (roughly, we would need “no unmeasured moderators correlated with norms”)





1 IQR = 0.6 extra problems
on worksheet task
Increase
+0.5 IQR
+1 IQR
+10%
Orig
Group
Low Norm/Lower Ach
High Norm/Lower Ach
Original 0.032 (-0.011 0.076)
+10%   0.050 (0.005, 0.097)
+Half IQR 0.051 (0.005, 0.099)
+Full IQR 0.059 (0.009, 0.114)
density
0
5
10
15
0.0
0.1
0.2
CATE

Conclusion

- Flexible models + careful regularization + posterior summarization is a winning combination
- Our approach takes the best parts of linear models with lots of researcher degrees of freedom and “black box” machine learning methods that only afford bankshot regularization and summarization
    - Many “degrees of freedom” in the summarization step, but these depend on the data only through the posterior
    - Unlike many ML methods, we can handle multilevel structure and prior knowledge with ease

$$y_i = f(\mathbf{x}_i) + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2)$$

$$f(\mathbf{x}) = \sum_{h=1}^{m} g(\mathbf{x}, T_h, M_h)$$